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Sensitivity of Open-Loop Typical Section Gust Response to Structural Parameters

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Introduction

THE alleviation of the gust response of aircraft has been a topic of research for a number of years.[1] The goals of this research have been 1) to reduce the rms values of the loads that the aircraft experiences from a gust, 2) to increase fatigue life, 3) to better design the structure, 4) to provide increased comfort for the passengers and crew, and 5) to decrease the effect of the gust on the cargo.[1] One approach to gust alleviation is a passive approach, where an existing structure is resized to alleviate gust loads or to minimize the structural mass when subjected to a given gust.[2,3] The most common approach has been to include active control systems in the design.[4]

However, to the best of the author's knowledge, except for the very preliminary results of Ref. 3, there has not been an effort toward understanding the effects of structural modifications on the gust response of the wing with or without a control system. More significantly, there has been no research into understanding or quantifying the magnitude of the change in the open-loop gust response of a wing caused solely by structural modifications, nor into quantifying the sensitivity of the gust response caused by structural modifications. In particular, could a pure structural redesign significantly reduce the gust response? This Note seeks to begin to examine these issues by using a typical section model.

This Note uses the rms response as a measure of the gust response. This was done because it is a convenient and physically meaningful measure of the system response to a random input (the gust). For past aeroelastic research efforts, the rms has been used only for examining closed-loop aeroservoelastic systems. The rms approach has not been used to determine the influence of solely the structure on the gust response (open-loop), nor to quantify to what extent structural modifications can change the open-loop gust response of the aeroelastic system. Moreover, past researchers have typically examined the sensitivity of the weighted control cost, not the output response for pitch and plunge.

In this Note, the rms response of a typical section model subjected to a gust is examined using the state covariance matrix. In addition to determining the rms, this Note will use it as a guide for understanding (characterizing) the gust behavior of aeroelastic systems. The sensitivity of the rms behavior with respect to several aeroelastic parameters will be examined to gain some insight into passive aeroelastic stability augmentation design for gust alleviation. As a result of the characterization of the gust response sensitivity, the usefulness of the state covariance matrix for passive aeroelastic tailoring for gust response will be demonstrated.

It is hoped that from the characterization a range of questions can begin to be addressed. Is there one characteristic parameter or a small group of parameters that may be used to assess the full range of design possibilities over a range of airspeeds? In addition, if a parameter or combination of parameters identifies a design with a desired gust response, can it be extended to similar planforms? What, if any, are the limitations to reducing the gust response by inertial and structural redesign? This Note will begin to examine these questions by approaching the gust response as a mechanical dynamic response whose underlying cause is the aeroelastic interaction between characteristic deformations of the typical section.

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Model

This Note uses a typical section model that is quite well documented in the literature.[4] Although it is a simple model, and in some sense an educational model, it retains the major elements of aeroelastic behavior and does allow for insight into the fundamental behavior of aeroelastic models. Its simplicity allows for some characterization of general behavior that would not be practical for a more complex model.

Determination of the System RMS

The rms provides a convenient, scalar measure of the energy that the disturbance injects into the different modes of motion of the system. Moreover, it is a useful, physically meaningful measure of the response of a system to a random disturbance. Furthermore, it is a relatively easy quantity to compute for linear time-invariant systems.

Consider a linear time-invariant state–space model of an aeroelastic system

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + G\boldsymbol{w}, \qquad \boldsymbol{y} = C\boldsymbol{x} \tag{1}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ is the control input vector, $\boldsymbol{w}$ is the vector of disturbances, and $\boldsymbol{y}$ is the vector of system outputs. In general, provided that the plant is observable and controllable, stability is proven by finding a Lyapunov function for the system. For linear time-invariant systems, such as those described by Eq. (1), stability is then conditioned on the existence of a positive definite solution X to the following Lyapunov equation:

$$AX + XA^T + GWG^T = 0 \tag{2}$$

where A is the plant matrix, W is the intensity matrix of the disturbance (in this case the gust), and G describes how the disturbance affects the plant. X is the system covariance matrix. The system covariance matrix is related to the output covariance matrix Y by the following:

$$Y = CXC^T \tag{3}$$

where the matrix C is termed the output matrix in a first-order state–space model, and Y is n_y by n_y. The diagonal elements of Y are the mean square outputs of the system

$$\sigma_i^2 = [CXC^T]_{ii}, \qquad i = 1, \ldots, n_y \tag{4}$$

where σ_i^2 are the mean square outputs and σ_i is the rms measure of the system outputs. Because this is an open-loop model, by judiciously choosing the output matrix, the rms of any desired state can be easily computed. In the case of a modal model, the same situation is true, but involves the eigenvector matrix of the structural system in the C matrix.

Computing the sensitivity of the rms with respect to a scalar design parameter p is also a straightforward task. First, applying the chain rule to Eq. (4) results in

$$\frac{\partial(\sigma_i)}{\partial p} = \frac{1}{2}\,\frac{\left[C\,\dfrac{\partial X}{\partial p}\,C^T\right]_{ii}}{\sqrt{[CXC^T]_{ii}}}, \qquad i = 1, \ldots, n_y \tag{5}$$

In the general case, C could be a function of p, but in this Note, C is chosen to be constant with respect to p. For a modal model, C is also a function of the structural design parameter p because the eigenvector matrix will be a function of p. Computing the gradient of the Lyapunov solution $\partial X/\partial p$ again is straightforward, by applying the chain rule to Eq. (2) and realizing that the result is also a Lyapunov equation in $\partial X/\partial p$

$$A\,\frac{\partial X}{\partial p} + \frac{\partial X}{\partial p}\,A^T + \left[\frac{\partial A}{\partial p}\,X + X\left(\frac{\partial A}{\partial p}\right)^T\right] = 0 \tag{6}$$

This equation assumes that D and W are independent of p, and therefore their gradients are zero.

Design Studies

The model parameters and their nominal values for the typical section are given in Ref. 4. They are the same as those used in Ref. 4. In the studies, except where noted, all model parameters assume their nominal value as in Ref. 4. The approach to the design studies is to vary important aeroelastic parameters and compute the sensitivity of the rms over a range of nondimensional airspeeds U. This Note focuses on the aft shear center case (wash-in). The parameters that were chosen are the shear center location A_e, the ratio of bending frequency-to-torsional frequency R, and mass center x_α. From the resulting studies, some general characterization of the gust response and its sensitivity for the model will be made.

The sensitivity of the gust response is shown in Figs. 1–3. Because R is a dominant parameter in the response, the sensitivities will be presented vs U for three values of R, 0.25, 0.5, and 1.

Figure 1 plots the sensitivity of the pitch and plunge rms to A_e vs U for the aft shear center (wash-in) $A_e = 0.4$ case, and for the three values of R. The sensitivities for the pitch and plunge are of similar value and behavior to each other. They are both positive over the entire range of nondimensional airspeeds, with small values away from the instability speed. This large variation in the sensitivity is not unexpected. Figure 1 indicates the large effect the shear center has on the gust response. Also note that, as the instability speed is approached, the rms response goes to infinity.

Fig. 1 Sensitivity of pitch and plunge response rms to shear center location.

Fig. 2 Sensitivity of pitch and plunge response rms to *R*.

Fig. 3 Sensitivity of pitch and plunge response rms to x_α.

Figure 2 plots the sensitivity of the pitch and plunge rms to R vs U for the aft shear center and for the three values of R. The sensitivity of the gust response for the pitch and plunge rms are of a relatively similar value to each other for the same value of R. However, the sign of the sensitivity to R is opposite to that of the shear center (although of similar magnitude). As the nondimensional airspeed is increased, the sensitivity is first positive, then changes to a large negative value. This is generally true for all three values of the frequency ratio.

Figure 3 presents the sensitivity of the response to x_α as a function of U for the three values of R. Again, the sensitivities are of similar value. However, this figure illustrates the wide variation in sign that can occur for the sensitivities. The sensitivity of both the pitch and plunge rms is at first negative for low values of U, then increases, eventually becoming positive as the instability speed is approached.

Observations and Conclusions

This Note has sought to gain insight into the gust response of a wing by examining the gust behavior of a typical wing section model. By using the state covariance matrix, the rms of the response can be easily determined. The sensitivity of the gust response for the shear center location, for variation in R, and for mass center was computed.

The group of parameters chosen in this Note have a great impact on the gust response of the typical section model. Not shown in this Note are the actual pitch and plunge rms responses for the problem. Away from the instability speed, the response rms is generally of the order of $10x^{-2}$. As shown in Figs. 1–3, the sensitivity is of order 1. Therefore, the sensitivities are approximately 3 orders of magnitude larger than the actual response. In addition, the sensitivity of the gust response can vary in sign over the range of nondimensional airspeeds examined here. Consequently, if one were to try to use aeroelastic tailoring for minimizing the gust response, extreme care would have to be taken in approach. In particular, if an optimization scheme were to be used, it would have to be fairly robust to allow for wide variations in the objective function (the gust response). In addition, one could use these sensitivities as guides for a preliminary examination (parameter studies or trade studies) of designing the wing for the gust response.

This Note has laid the groundwork for a better understanding of how the rms gust response and its sensitivity vary with airspeed and structural parameters. It has shown that the sensitivity of the gust response is much larger than the actual response by approximately 3 orders of magnitude. Consequently, the opportunity for aeroelastic tailoring for gust response has been demonstrated, but care must be taken when performing the tailoring.

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Thrust Offset Effect on Longitudinal Dynamic Stability

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Nomenclature

A = aspect ratio
C_D = drag coefficient
$C_{D_{\min}}$ = minimum or parasitic drag coefficient
C_L = lift coefficient
C_{L_α} = $\partial C_L/\partial\alpha$
C_m = pitching moment coefficient
C_{m_α} = $\partial C_m/\partial\alpha$
$\bar{c}$ = mean aerodynamic chord
D = drag
e = efficiency factor for wing-induced drag
g = acceleration caused by gravity
I_Y = moment of inertia of aircraft about y axis
K_n = static margin, $-C_{m_\alpha}/C_{L_\alpha}$
L = lift
M = pitching moment about the c.g.
$M_{q,u,w,\dot{w}}$ = $(\partial M/\partial q)/I_y$, $(\partial M/\partial u)/I_y$, $(\partial M/\partial w)/I_y$, $(\partial M/\partial \dot{w})/I_y$
m = aircraft mass
$Oxyz$ = axes fixed in aircraft
q = angular velocity in pitch
S = wing area
T = thrust
t = time
u = velocity component in x direction
u_0 = reference flight velocity
W = aircraft weight
w = velocity component in z direction
X = force component along $0x$
$X_{u,w}$ = $(\partial X/\partial u)/m$, $(\partial X/\partial w)/m$
Z = force component along $0z$
$Z_{u,w}$ = $(\partial Z/\partial u)/m$, $(\partial Z/\partial w)/m$
z_{TH} = perpendicular distance of thrust line below aircraft c.g.
α = angle of attack
Δh_n = neutral point shift because of thrust line offset determined from trim-slope criterion
Δu = perturbation of aircraft velocity component in x direction
Δw = perturbation of aircraft velocity component in z direction
$\Delta\theta$ = pitch deviation angle
λ = root of stability equation
ρ = density of air

Subscript

0 = reference flight

Introduction

THRUST lines of action that pass above or below the c.g. affect both the trim and stability of an aircraft. Examples include flying boats with high-mounted engines or transport aircraft with engines mounted on pylons below a low wing. Direct thrust effects on trim are relatively straightforward to evaluate, but stability changes with power require more consideration. The definition of the neutral point has caused some confusion. Standard flight tests rely on elevator deflections measured at different trimmed flight speeds for a range of c.g. positions. The neutral point is then defined as the c.g. position at which the gradient of the elevator deflection to trim vs lift coefficient is zero. For a constant-thrust jet engine this leads to a neutral point shift Δh_n caused by thrust offset given by Solies[1] as

$$\Delta h_n = -(T/W)(z_{TH}/\bar{c}) \tag{1}$$

A low-mounted engine with $z_{TH} > 0$ is considered as destabilizing, whereas a high-mounted engine with $z_{TH} < 0$ is stabilizing. However, Etkin[2] states that the application of the trim-slope criterion can be misleading as to stability and defines the neutral point on the basis of the pitch stability term $C_{m_\alpha}(=\partial C_m/\partial\alpha)$. A similar view is expressed by Solies.[1]

To obtain a clearer understanding of thrust offset effects it is necessary to consider dynamic stability by solving the usual linearized, longitudinal equations of motion.

Equations of Motion

For initial horizontal flight and neglecting the aerodynamic derivatives $Z_{\dot{w}}$ and Z_q, Nelson[3] gives the following linearized, longitudinal equations of motion:

$$\begin{aligned} \left(\frac{d}{dt} - X_u\right)\Delta u - X_w\Delta w + g\Delta\theta &= 0 \\ -Z_u\Delta u + \left(\frac{d}{dt} - Z_w\right)\Delta w - u_0\frac{d}{dt}\Delta\theta &= 0 \\ -M_u\Delta u - \left(M_{\dot{w}}\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\theta &= 0 \end{aligned} \tag{2}$$

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